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ELEMENTARY MATHEMATICS TEACHERS' SPECIALIZED CONTENT KNOWLEDGE RELATED TO DIVISION BY ZERO

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ABSTRACT

Division by zero is often unclear and confusing, and teachers at various grade levels may encounter difficulties in conveying this concept to their students. The concept of dividing by zero plays a crucial role in developing an understanding of certain mathematical concepts, such as rational numbers and the relationship between multiplication and division. Therefore, in order to provide students with a strong conceptual understanding about division by zero, it is important to examine the in-service teachers' knowledge. The aim of this study was to explore the in-service teachers' specialized content knowledge concerning division by zero, as well as the effects of variables which are years of service and education status on teachers' specialized content knowledge. The study consisted of a case-study and was carried out with 82 in-service teachers of elementary mathematics at Afyonkarahisar in Turkey. To determine the in-service teachers' specialized content knowledge, a written questionnaire was used. Data were analyzed by using inductive and deductive analysis. Analysis of the results revealed that although most of the in-service teachers gave correct answers related to division by zero, few of them provided conceptual explanations. Rather, those who gave correct explanations mainly responded with rule-based statements.

Keyword: *Specialized Content Knowledge, Subject Matter Knowledge, Division by Zero*

INTRODUCTION

Division by zero is often unclear and confusing, and teachers at various grade levels may encounter difficulties in conveying this concept to their students. The concept of dividing by zero plays a crucial role in developing an understanding of certain mathematical concepts, such as rational numbers and the relationship between multiplication and division (Quin, Lamberg, & Perrin, 2008). Therefore, in order to provide students with a strong conceptual understanding about division by zero, it is important to examine the in-service teachers' knowledge. Over the last four decades, researchers are increasingly focusing on the content knowledge of teachers and its role in teaching. In research on the teaching of mathematics, several researchers examined teachers' beliefs and attitude about some mathematical concepts (Dede & Karakuş, 2014; Thompson, 1992; Zikre & Kwan Eu, 2016). These studies show that teachers' beliefs and attitude affect their teaching. Other researchers focused on teachers' and pre-service teachers' understandings of some mathematical concepts such as zero, fraction or decimal numbers (Baki, 2013; Ball, 1990; Even, 1993; Even & Tirosh, 1995; Gökkurt, Şahin & Soylu, 2012; Toluk-Uçar, 2011). They investigate how teachers think about their mathematical

knowledge and how they explain and justify their mathematical ideas. Ball, Thames, and Phelps (2008) developed a model called "mathematical knowledge for teaching (MKT)" for understanding teacher knowledge in the field of mathematics education. In this study, it was intended to assess the in-service teachers' special content knowledge based on the Ball et al. (2008) framework.

Mathematical Knowledge for Teaching

Teachers' knowledge is one of the most important factors relating to the quality of teaching. For over 40 years, researchers have examined teachers' knowledge and suggested some models on it. One of the most commonly used and widely accepted model of teachers' knowledge was given by Shulman (1986) who divided teacher content knowledge into three categories which are subject matter knowledge (SMK), pedagogical content knowledge (PCK) and curricular knowledge (CK). Shulman's (1986) model has been accepted as a framework for most of the researches on teacher knowledge (Ball, 1990; Ball & McDiarmid, 1990; Ball, Thames & Phelps, 2008). Ball et al. (2008) improved Shulman's (1986) model and developed a framework for mathematics teaching. The focus of their framework was all the things that teachers do in their mathematics teaching. All the things refer to having a strong knowledge of the subject matter, choosing appropriate examples and representations, identifying students' errors and misconceptions, examining the sources of these misconceptions, being aware of the students' preliminary knowledge and using mathematical language correctly (Hill, Rowan & Ball, 2005). In other words, it refers to what teachers need to do in teaching mathematics called "mathematical knowledge for teaching (MKT)" (Ball et al., 2008). Ball et al. (2008)'s purpose in developing MKT was not to replace Shulman's model, but instead to provide further understanding of teachers' knowledge by building on it. MKT is comprised of two halves, each of which includes three components. These components can be categorized under Shulman's two types of knowledge, PCK and SMK (refer to Figure 1).

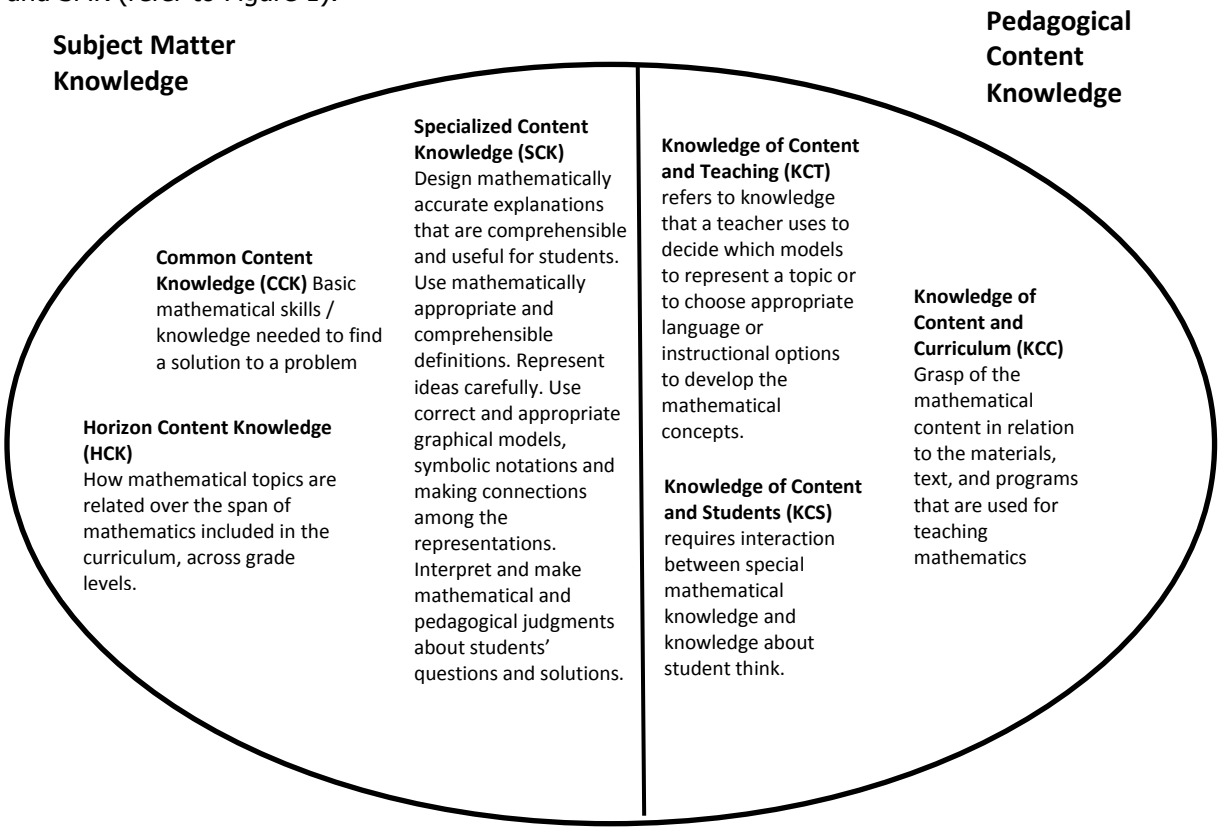


Figure 1. Domains of mathematical knowledge for teaching. Adapted from Bair and Rich (2011) and Ball et al. (2008).

Specialized Content Knowledge (SCK) is a specific knowledge used for teaching that someone other than a teacher does not need to possess (Aslan-Tutak, 2009). Ball et al. (2008) explained this idea as "accountants have to calculate and reconcile numbers and engineers have to mathematically model properties of materials, but neither group needs to explain why, when you multiply by 10, you 'add a zero" (p. 401). SCK involves teacher knowledge about how to explain and justify a student's mathematical ideas and how to choose, make and use mathematical models and representations. For example, when a teacher explains and justifies why you invert and multiply to divide fractions, the teacher uses specialized content knowledge.

Research on Teachers' Knowledge of Division by Zero

In the literature, there are some studies investigating teachers' and pre-service teachers' knowledge based on MKT (Aslan-Tutak, 2009; Burke, 2013; Contreas, Batanero, Diaz & Fernandes, 2011; Nolan, Dempsey, Lovatt & O'Shea, 2015). These studies indicate that teachers' mathematical content knowledge and pedagogical content knowledge has a direct influence on their mathematics teaching. Teachers design mathematically accurate activities, pose good mathematical problems and make appropriate instructional materials and models based on this knowledge. The mathematical content knowledge is directly affected by teacher's pedagogical content knowledge. Brown and Borko (1992) expressed that limited mathematical content knowledge is an obstacle on pre-service teachers' pedagogical content knowledge. Even (1993) found that many of the pre-service teachers in her study lacked the necessary subject matter knowledge. This fact had affected their pedagogical content knowledge. There must be a tight connection between pedagogical content knowledge and subject matter knowledge for it to not be an obstacle instead. Many mathematics teachers have low levels of the SCK and PCK required to teach the subject effectively (Tirosh, 2000; Ball, 1990). The concept of division by zero is one of the important topics in mathematics that pre-service teachers often seem to have insufficient knowledge. Ball (1990) studied elementary and secondary pre-service teachers' specialized mathematical knowledge of division by zero and found that pre-service teachers had significant difficulty with the meaning of division by zero. Ball (1990) expressed that pre-service teachers mostly gave rule-based explanations such as "it is undefined because this is a rule, and you cannot divide by zero". Likewise, Wheeler and Feghali (1983) reached a similar conclusion, reporting that pre-service elementary school teachers did not have an adequate understanding of the number zero or the division of items with zero as a dividend or a divisor. Crespo and Nicol (2006) also examined the effects of two teacher education tasks on pre-service teachers' understanding of division by zero. According to their findings, the participants made progress during these instructional experiences; as such, their initial ideas were extended, and their later explanations became more conceptually based than rule-bound.

Beside pre-service teachers, there are some studies in which in-service teachers' knowledge have also been discussed (Cankoy, 2010; Even & Tirosh, 1995; Quinn et al., 2008). For instance, Even and Tirosh (1995) asked secondary mathematics teachers to explain why 4 divided by 0 is undefined. They found that most of the teachers could not supply any appropriate explanation. Similar to Ball's (1990) findings, some teachers provided rule-based explanations such as "in mathematics this is a rule that one cannot divide by zero". Likewise, Quinn et al. (2008) investigated the perceptions of fourth through eighth grade teachers concerning division by zero and reported that, although a few teachers had a conceptual understanding, most did not. In addition, Cankoy (2010) investigated high-school mathematics teachers' topic-specific pedagogical content knowledge about "$a^0=1$, $0!=1$ and $a\div0$, where $a\neq0$." He found that experienced teachers proposed conceptual explanations more frequently than novice teachers, but all the participants' explanations were procedural and fostered memorization.

Division by Zero in the Turkish Educational Setting

In Turkey, students begin learning about division in the second grade. However, neither the mathematics teaching program nor the authorized textbooks offer much explanation about the fact that a number cannot be divided by zero. Students first begin to learn about rational numbers in the

7th grade. At this level, textbooks define rational numbers as "can be written as a/b where a and b (b≠0) are integers" (Bağcı, 2015, p. 37). According to this definition, numbers such as $-\frac{5}{2}, \frac{0}{5}, \frac{3}{8}$ are considered as rational numbers, since their denominators and dividends are integers. On the other hand, although their denominators and dividends are also integers, numbers such as $\frac{4}{0}, \frac{7}{0}$ are not considered as rational numbers, because their denominators are zero (Bağcı, 2015). This textbook, as well as others, presents $\frac{0}{0}$ as undetermined but there is no explanation as to why this is the case. Then, in the 9th through 12th grades, students frequently encounter such undefined and undetermined cases while solving equations, as well as in defining functions and finding limits of functions. As with the 7th grade, rather than offering conceptual explanations related to division by zero, the focus on solving this undefined case is on the rules and procedural operations.

PURPOSE OF THE STUDY AND RESEARCH QUESTIONS

The aim of this study was to explore the in-service teachers' specialized content knowledge of dividing by zero. Therefore, the research problems of this study were as follows:

1. What are the elementary in-service mathematics teachers' specialized content knowledge of division by zero?
2. Does the years of service have any effect on the elementary in-service mathematics teachers' specialized content knowledge of division by zero?
3. Does the education status have any effect on the elementary in-service mathematics teachers' specialized content knowledge of division by zero?

METHODOLOGY

The research methodology of this study was a case study. In a case study, the researcher is primarily focused on understanding a specific individual or situation (Fraenkel, Wallen, & Hyun, 2012). Case study research focuses on individuals' experiences of certain phenomenon and describes the cases in depth.

Participants

Data were collected from the 82 in-service elementary mathematics teachers at the end of the spring semester of the 2016-2017 academic year. In order to investigate the in-service teachers' SCK of division by zero, in-service teachers in city center of Afyonkarahisar, Turkey were selected. Teachers with different years of service and education status were purposively selected based on certain criteria. The aim in selecting in-service teachers from different years of service and education status was to compare the novice and expert teachers' explanations and to investigate the differences in teachers' explanations depending on the experiment. Demographic characteristics of 82 in-service teachers were given in Table 1.

Table 1
Demographic characteristics of in-service teachers

		f	%
Years of service	1-5 years	21	26
	6-10 years	24	29
	11-15 years	26	32
	16-20 years	11	13
Total		82	100
Education status	License degree	68	83
	Master degree	14	17
Total		82	100

A pre-service elementary mathematics teacher graduates from teacher education program at the end of four years with teaching degree in Turkey. Depending the result of teacher proficiency exam, the pre-service mathematics teacher starts to work as a mathematics teacher. Moreover, the pre-service teachers can arbitrarily get master education after working as a mathematics teacher.

Instrument and Data Collection Procedure

To determine the in-service teachers' SCK, a written questionnaire was developed. The first questionnaire item was, "Suppose that a student asks you what 7 divided by 0 is. How would you respond?" The second questionnaire item was, "Suppose that a student asks you what 0 divided by 0 is. How would you respond?" The aim of these questions was to reveal the in-service teachers' SCK concerning division by zero. The questions were adapted from the literature relating to teachers' knowledge and perceptions about division by zero (Ball, 1990; Cankoy, 2010; Quinn et. al, 2008). The questionnaire was administered to the participants in the spring term of the 2016-2017 academic year. The in-service teachers were given sufficient time to complete their responses. Before administering the final form of the questionnaire, two mathematics educators checked the validity of the questions and agreed that they were valid and appropriate for measuring in-service teachers' SCK.

Data analysis

After the instrument was administered to 82 in-service teachers, the researchers and two mathematics educators chose the explanations of 25 of the participants at random and used inductive and deductive analysis (McMillan & Schumacher, 2006) to determine the categories and themes in the written responses. The qualitative analysis of the data led to the development of four categories of explanation. Ball's (1990) model explained the understanding of dividing by zero guided of coding. The first category included in-service teachers who gave accurate explanations that are comprehensible for students and provided meaning for students. Explanation of in-service teachers who provided meaning were separated into five sub-categories in the form of "inverse of multiplication", "sharing/dividing evenly", "intuitive notion of limit", "part-whole comparisons" and "deductive reasoning". The second category included in-service teachers who knew that the operation is undefined/undetermined, but could not provide any explanation as to why this is the case. Explanations of in-service teachers who gave the correct rule were separated into three sub-categories in the form of "explain as a rule", "the meaning of zero", and "definition of rational number". The third category included in-service teachers who gave incorrect explanation. Explanation of in-service teachers who provided incorrect answers were separated into two sub-categories in the form of "explain as an infinite", and "write a number". The last category included in-service teachers who did not give any explanation. All teachers' names were covered and added codes such as T1, T2, … in order to identify them. Aside from that, direct quotations were drawn from teachers' explanations when the data were presented.

FINDINGS

In-service Teachers' SCK Related to 7÷0 with regard to years of service

The frequencies and percentages of the in-service teachers' explanations on 7÷0 with regard to years of service are given in Table 2.

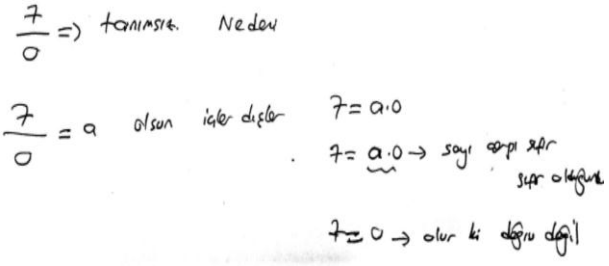
Table 2
Frequencies and percentages of in-service teachers' explanations on 7÷0 with regard to years of service

Category	Theme		1-5 years	6-10 years	11-15 years	16-20 years	Total
Meaning	Inverse of multiplication	f	6	11	12	9	38
		%	7	13	15	11	47
	Sharing/ dividing evenly	f	2	2	3	-	7
		%	2	2	4	-	9
	Intuitive notion of limit	f	1	-	2	-	3
		%	1	-	2	-	3
	Part-whole comparisons	f	-	-	2	-	2
		%	-	-	2	-	2
Correct rule	Explain as a rule	f	2	5	5	2	14
		%	2	6	6	2	17
	The meaning of zero	f	3	4	1	-	8
		%	4	5	1	-	10
	Definition of rational number	f	2	-	-	-	2
		%	2				2
Incorrect rule	Explain as infinite	f	4	2	1	-	7
		%	5	2	1		9
	Write a number	f	1	-	-	-	1
		%	1	-	-	-	1

Table 2 shows that nearly 61% of the in-service teachers explained the meaning of division by zero, while 39% of them responded by stating rules, ten of which were incorrect. Four approaches were used on explanations focused on meaning: (1) inverse of multiplication, (2) sharing/dividing evenly (3) intuitive notion of limit and (4) part-whole comparisons.

T28 who is a teacher with 1-5 years of service chose the inverse of multiplication approach:

Original explanation

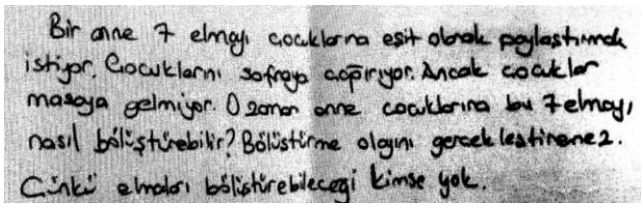


Translation from Turkish to English

7/0 is undefined, because, if 7/0=a then we can write 7=0.a. If we multiply a number by zero, the result is always zero. Clearly, there is no number that can be placed in a to make the equation true.

T54 who is a teacher with 11-15 years of service chose the second approach:

Original explanation

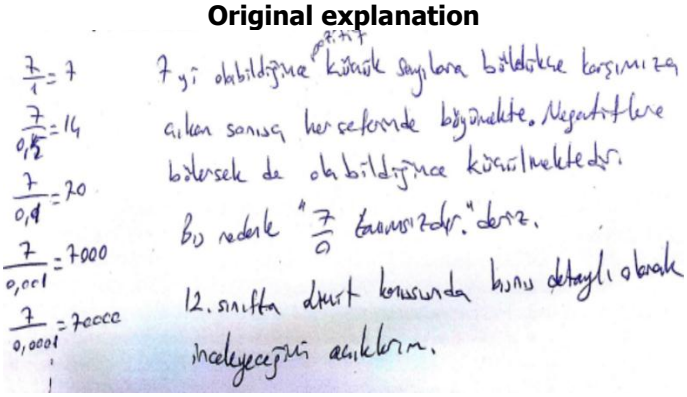

Bir anne 7 elmayı çocuklara eşit olarak paylaştırmak istiyor. Çocuklarını sofraya çağırıyor. Ancak çocuklar masaya gelmiyor. O zaman anne çocuklara bu 7 elmayı nasıl bölüştürebilir? Bölüştürme olayını gerçekleştirene2. Çünkü elmaları bölüştürebileceği kimse yok.

Translation from Turkish to English

A mother share 7 apples with children evenly. She call their children to the dining table. Yet, the children are not coming. There are no children to share apples, how can she divide 7 apples between zero children? Not having any children makes division impossible.

T19 who is a teacher with 11-15 years of service chose the third approach:

Original explanation

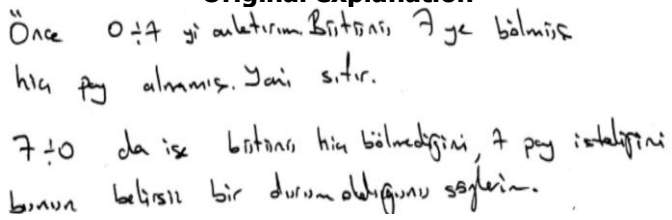


Translation from Turkish to English

7÷1=7, 7÷0.5=14, 7÷0.1=70, 7÷0.001=7000, 7÷0.0001=70000, …
If we decrease the dominator, the result increases. If the denominator gets closer to zero result will be very huge. For that reason 7÷0 is undefined. This topic will be explained more deeper in grade 12 in the concept of limit

T27 who is a teacher with 11-15 years of service chose the fourth approach:

Original explanation



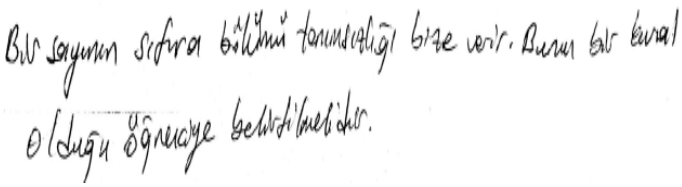
Translation from Turkish to English

First of all, I will explain the 0÷7. A whole is cut into seven parts, but no part taken. So, it is zero.
In the case of 7÷0, the whole is not cut any part, but seven parts were taken. I will explain it is undefined.

Table 2 shows that 29% of in-service teachers explained division by zero in terms of a rule such as "7÷0 is undefined, because this is a rule in mathematics." Unlike those who focused on meaning, these in-service teachers did not try to show why this was so. Three approaches were used division by zero in terms of a rule: (1) explain as a rule, (2) the meaning of zero and (3) definition of rational number.

T18 who is a teacher with 1-5 years of service chose the first approach:

Original explanation

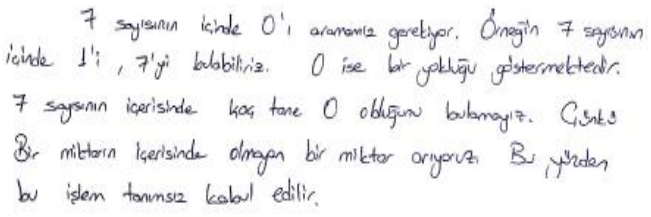


Translation from Turkish to English

A number divided by zero is undefined. It should be emphasized that this is a rule.

T40 who is a teacher with 1-5 years of service chose the second approach:

Original explanation



Translation from Turkish to English

We should know how many zero are there in 7. For example, There are seven union groups in 7. But, zero means nothing. So, we cannot find how many zero are there in 7. Because, we can group nothing into something that exits. For that reason this operation is accepted as undefined.

T49 who is 1-5 years of service chose the third approach:

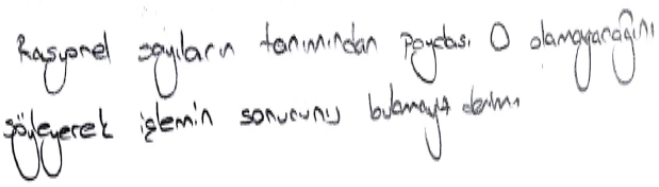
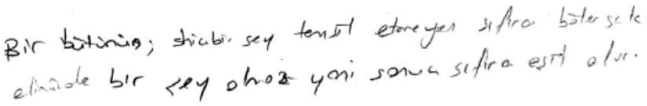
Original explanation	Translation from Turkish to English
	By the definition of rational numbers, the denominator cannot be zero. So we cannot calculate this operation.

Table 2 shows that unlike those who gave correct rule-based explanations, 10% of in-service teachers gave incorrect answer or misleading explanations on division by zero. There were two incorrect explanations on division by zero: (1) explain as an infinite and (2) write a number.

T23 who is 11-15 years of service chose the first incorrect explanation:

Original explanation	Translation from Turkish to English
	7/0=∞, I express this is a rule.

T48 who is 1-5 years of service chose the second incorrect approach:

Original explanation	Translation from Turkish to English
	if we divide a whole into zero that do not represent anything, then nothing happens, so the result is zero

According to Table 2, the most frequently given explanation in the explanations focusing on meaning were the "inverse of multiplication" at all years of service levels. Moreover, the in-service teachers with 11-15 years of service were mostly focusing on the meaning of division by zero and they gave more varied conceptual explanations. However, the least given explanations were "intuitive notion of limits" and "part-whole comparisons." Similarly, the most frequently given explanations in the correct rule category was the "explain as a rule" at all years of service levels. The second preferred correct rule explanation was the "meaning of zero." Moreover, only service teachers with just 1-5 years of experience gave explanations in the category of "definition of rational number". The in-service teachers who gave the most incorrect answers were in their 1-5 years of service.

The distribution of the in-service teachers' explanations on 7÷0 with regard to years of service is shown in Figure 2.

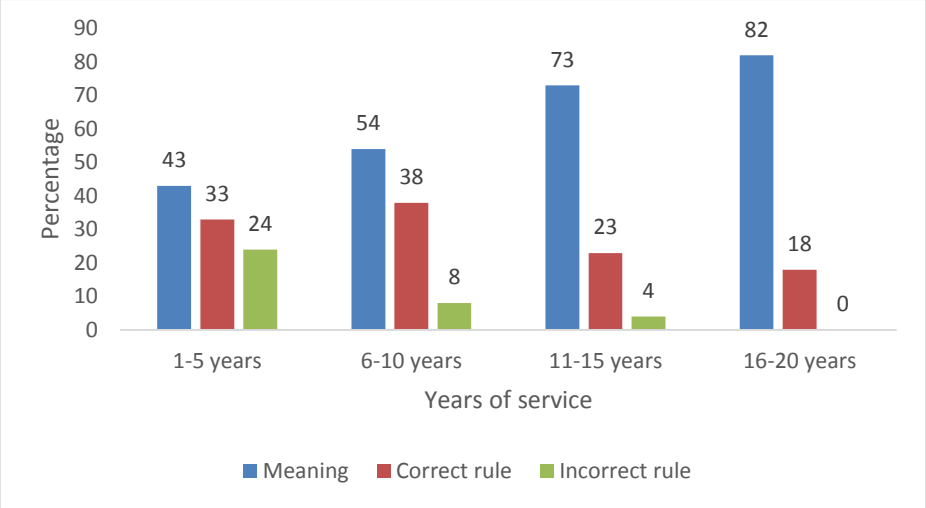


Figure 2. The distribution of the in-service teachers' explanations about 7÷0 with respect to years of service

As shown in Figure 2, the frequency of explanations focusing on meaning was high for in-service teachers in both 11-15 years and 16-20 years of service and there was an increase in the conceptual explanations from 1-5 years to 16-20 years of service. In addition, while there was a slight increase in the correct rule-based explanations in 6-10 years of service, there was a decrease in those rule-based explanations for the other years. Moreover, the number of incorrect explanations decreased from 1-5 years to 16-20 years.

The distribution of the in-service teachers' explanations on 7÷0 with regard to education status is shown in Figure 3.

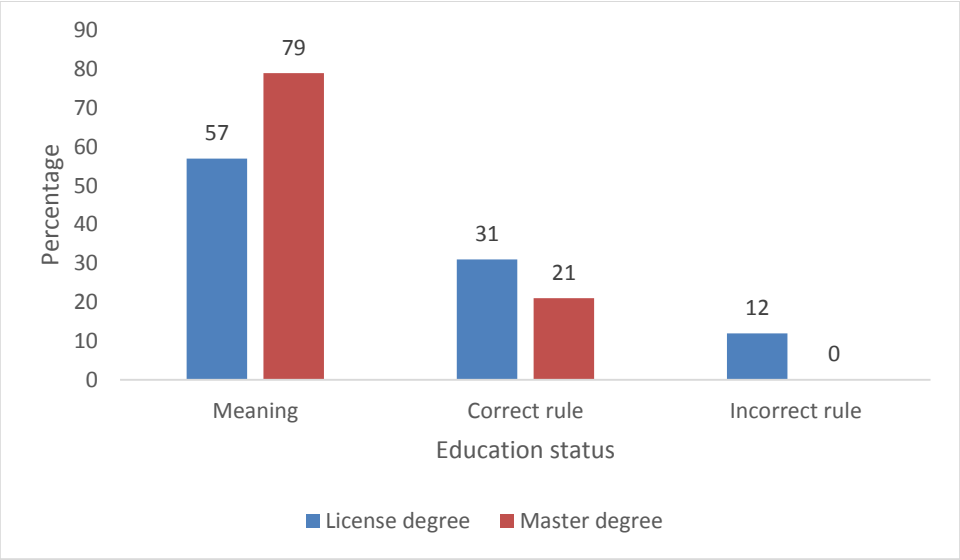


Figure 3. The distribution of the in-service teachers' explanations about 7÷0 with respect to education status

According to Figure 3, the percentage of master degree teachers' explanations focused on meaning was higher than license degree teachers. In contrast, the percentage of license degree teachers' both correct rule and incorrect explanations were higher than master degree teachers. This shows that education status has an effect on the teachers' explanations.

In-service Teachers' SCK Related to 0÷0 with regard to years of service

The frequencies and percentages of the in-service teachers' explanations on 0÷0 with regard to years of service are given in Table 4.

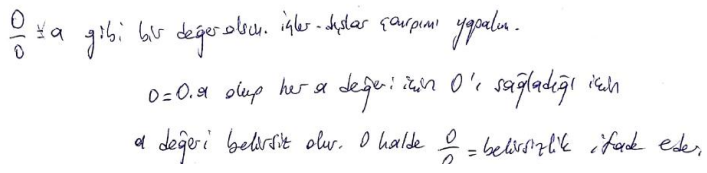
Table 4
Frequencies and percentages of in-service teachers' explanations on 0÷0 with regard to years of service

Category	Theme		1-5 years	6-10 years	11-15 years	16-20 years	Total
Meaning	Inverse of multiplication	f	7	12	8	5	32
		%	8	15	10	6	39
	Sharing/ dividing evenly	f	1	-	3	-	4
		%	1		3		5
	Deductive reasoning	f	-	1	-	-	1
		%		1			1
Correct rule	Explain as a rule	f	2	5	11	3	21
		%	2	6	13	3	26
	The meaning of zero	f	5	2	2	-	9
		%	6	2	2		11
	Definition of rational number	f	2	1	-	-	3
		%	2	1			4
Incorrect rule	Explain as infinite	f	2	2	2	1	7
		%	2	2	2	1	8
	Write a number	f	-	-	-	2	2
		%				2	2
No answer		f	2	1	-	-	3
		%	2	1			4

Table 4 shows that nearly 45% of the in-service teachers explained the meaning of 0÷0, while 51% of them responded by stating rules, and ten of which were found to be incorrect. Moreover, four in-service teachers did not give any explanations about 0÷0. Three approaches were used on explanations focused on meaning: (1) inverse of multiplication, (2) sharing/dividing evenly and (3) deductive reasoning.

T3 who is a teacher with 6-10 years of service chose the first approach:

Original explanation

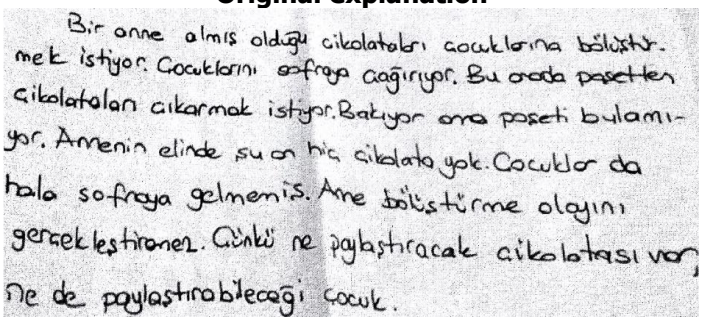


Translation from Turkish to English

Let's assume that 0/0=a. It would satisfy the equation 0=0a. In this equation, we can write any number as a; we cannot find a particular a. Thus 0/0 is undetermined.

T54 who is a teacher with 11-15 years of service chose the second approach:

Original explanation

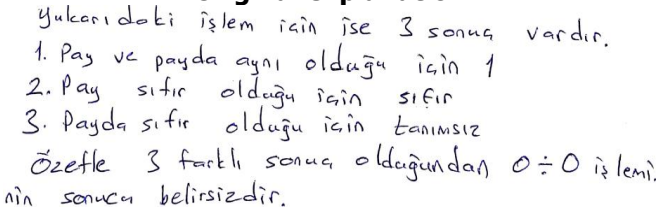

Bir anne almış olduğu çikolataları çocuklarına bölüştürmek istiyor. Çocuklarını sofraya çağırıyor. Bu arada poşetten çikolataları çıkarmak istiyor. Bakıyor ama poşeti bulamıyor. Annenin elinde şu an hiç çikolata yok. Çocuklar da hala sofraya gelmemiş. Anne bölüştürme olayını gerçekleştiremez. Çünkü ne paylaştıracak çikolatası var ne de paylaştırabileceği çocuk.

Translation from Turkish to English

A mother share chocolates with her children evenly. She wants to take off chocolates from the bag, but she cannot find the bag. Moreover, no children are coming. There are no chocolates and no children. Because there are no children or chocolates to share, mother cannot share evenly.

T50 who is a teacher with 6-10 years of service chose the last approach:

Original explanation


Yukarıdaki işlem için ise 3 sonuç vardır.
1. Pay ve payda aynı olduğu için 1
2. Pay sıfır olduğu için sıfır
3. Payda sıfır olduğu için tanımsız
Özetle 3 farklı sonuç olduğundan $0 \div 0$ işleminin sonucu belirsizdir.

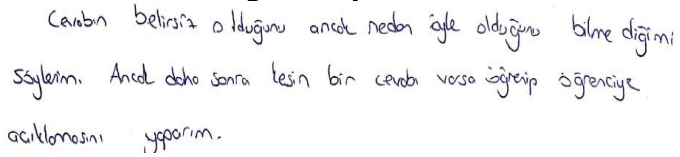
Translation from Turkish to English

The above calculation has three results
1. Since the numerator and the denominator are both same, result is 1
2. Since the numerator is zero, the result is zero
3. Since the denominator is zero, the result is undefined
In sum, since there are three different results, the result of calculation should be undetermined.

Table 4 shows that 41% of in-service teachers explained division by zero in terms of a rule such as "0÷0 is undefined, because this is a rule." Unlike those who focused on meaning, these in-service teachers did not try to show why this was so. Three approaches were used division by zero in terms of a rule: (1) explain as a rule, (2) the meaning of zero and (3) definition of rational number.

T37 who is a teacher with 1-5 years of service chose the first approach:

Original explanation

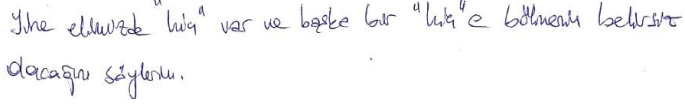

Cevabın belirsiz olduğunu ancak neden öyle olduğunu bilmediğimi söylerim. Ancak daha sonra kesin bir cevabı varsa öğrenip öğrenciye açıklamasını yaparım.

Translation from Turkish to English

I'd just say it is undetermined, but I don't know why this is so. If there is a define answer of this calculation, I will learn and explain to student.

T6 who is a teacher with 1-5 years of service chose the second approach:

Original explanation


Yine elimizde "hiç" var ve başka bir "hiç"e bölmenin belirsiz olacağını söylerim.

Translation from Turkish to English

We have "nothing" and I'd say dividing nothing to nothing is undetermined.

T44 who is a teacher with 1-5 years of service chose the last approach:

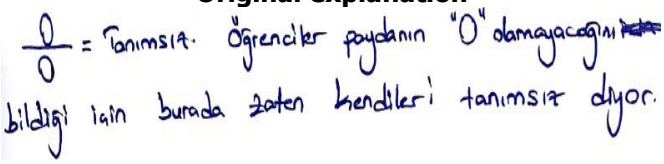
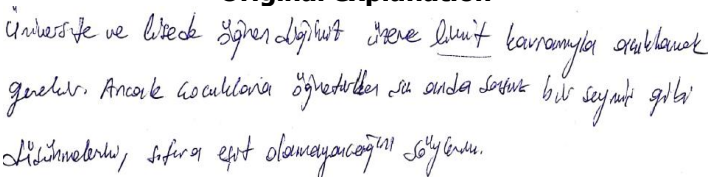
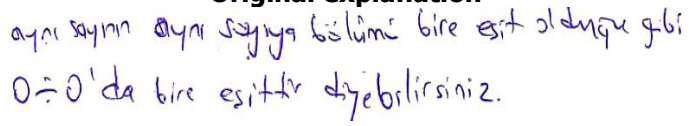
Original explanation	Translation from Turkish to English
	0/0 = undetermined. The students know that the denominator cannot be zero, they already express that it is undetermined.

Table 4 shows that unlike those who gave correct rule-based explanations, 10% of in-service teachers gave incorrect answer or misleading explanations on division by zero. There were two incorrect explanations on division by zero: (1) explain as an infinite and (2) write a number.

T18 who is a teacher with 1-5 years of service chose the first approach:

Original explanation	Translation from Turkish to English
	We have to explain with limit which we have learned in high school and university. But I'd say to students that the calculation result is infinite, it is not equal to zero.

T35 who is a teacher with 16-20 years of service chose the second approach:

Original explanation	Translation from Turkish to English
	As dividing a number to the same number is one, 0/0 is equal to one.

According to Table 4, the most frequently given explanation in the explanations focusing on meaning was the "inverse of multiplication" at all years of service levels. Moreover, in-service teachers in both 6-10 and 11-15 years of service were mostly focusing on the meaning of division by zero and they gave more varied conceptual explanations. However, the least given explanations were "deductive reasoning" and "part-whole comparisons." Similarly, the most frequently given explanations in the correct rule category was the "explain as a rule". The second preferred correct rule explanation was the "meaning of zero." In addition, most frequently given explanation in the category of incorrect rule was the "explain as infinite" at all years of service levels. There were also two in-service teachers in 16-20 years of service explained that the result should be a number. Only three teachers in both 1-5 and 6-10 years of service teachers gave no answer about 0÷0.

The distribution of the in-service teachers' explanations on 0÷0 with regard to years of service is shown in Figure 4.

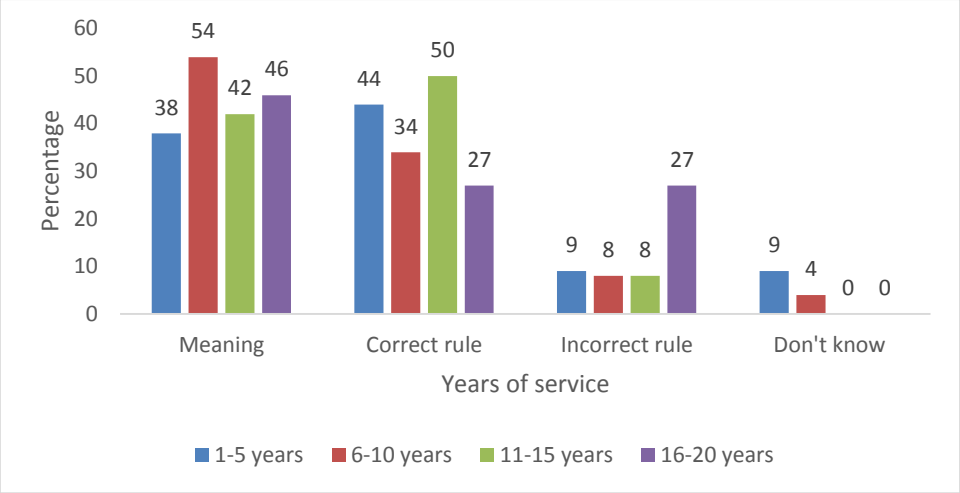


Figure 4. The distribution of the in-service teachers' explanations about 0÷0 with respect to years of service

As shown in Figure 4, the frequency of explanations focusing on meaning was high for in-service teachers in both 6-10 years and 16-20 years of service and there was an increase in the conceptual explanations from 1-5 years to 16-20 years of service. Also, while there was an increase in the correct rule-based explanations in 11-15 years of service, there was a decrease those rule-based explanations in 16-20 years of service. Contrary to that, the number of incorrect explanations is the highest in 16-20 years. Moreover, some teachers in both 1-5 and 6-10 years in service gave no any answer to division by zero.

The distribution of the in-service teachers' explanations on 0÷0 with regard to education status is shown in Figure 5.

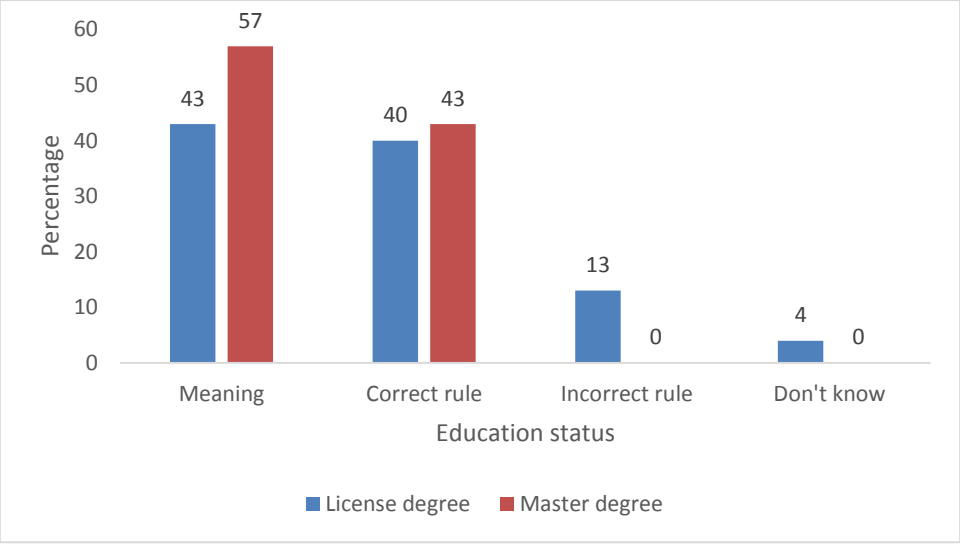


Figure 5. The distribution of the in-service teachers' explanations about 0÷0 with respect to education status.

According to Figure 5, the percentage of master degree teachers' explanations focused on meaning was higher than license degree teachers. Similarly, the percentage of master degree teachers' correct rule explanations were higher than license degree teachers. In contrast, the percentage of license degree teachers' incorrect rule explanations were higher than master degree teachers. This shows that education status has an effect on the teachers' explanations.

DISCUSSION

The results of this study demonstrate that most of teachers could give correct answers for both 7÷0 and 0÷0. Although most of the teachers provided correct answers related to division by zero, half of them had significant difficulty with the meaning of division by zero. The difficulties experienced by the teachers indicated a narrow understanding of division by zero. Studies reported that most of teachers knew that division by zero is undefined, but when asked to explain why, most could not supply any appropriate explanation (Cankoy, 2010; Even & Tirosh, 1995; Quin et al., 2008). Teachers who gave the meaning of division by zero correctly, mostly preferred "inverse of multiplication" strategy. However, few of them gave different strategies such as "intuitive notion of limit" and "deductive reasoning". This revealed that teachers had limited subject matter knowledge for teaching division by zero. Similar findings were observed by Ball (1990) and Cankoy (2010).

In comparing the responses according to years of service experienced teachers suggested more conceptual-based explanations for teaching division by zero than the novice teachers. Previous studies have reported that most of novice teachers do not have a solid understanding related to specific mathematical topics such as concept of zero, division and function (Even & Tirosh, 1995; Kinach, 2002; Ma, 1999; Toluk-Uçar, 2011). When the years of service increased, teachers conceptual-based explanations increased and became varied. This indicates that the years of service (experiment) impacted the teachers' specific content knowledge related to specific mathematical topics.

The results revealed that over half of teachers for teaching dividing by zero gave correct answer, while half of them responded by stating a rule. These teachers suggested explanations which promote procedural/rule-based understanding. Teachers who gave rule-based explanations mostly preferred "explain as a rule" and "the meaning of zero" strategies. Teachers in both 1-5 years and 6-10 years of service mostly gave rule-based explanations. Moreover, when the years of service increased, teachers' rule-based explanations decreased. Hutchinson (1996) notes that because novice teachers have weak pedagogical content knowledge, they prefer more rule-based explanations. So, teaching experience may be considered as an important factor in the development of teachers' specific content knowledge as seen in studies reporting that both teachers and pre-service teachers primarily gave rule-based explanations such as "this is a rule, you cannot divide by zero" to justify their answers (Ball, 1990; Cankoy, 2010; Crespo & Nicol, 2006; Quin et al., 2008). The result of this study showed that some teachers, especially novice teachers, provided incorrect explanations. Parallel to this result, studies (Ball, 1990; Charalambos et al., 2011; Kinach, 2002; Leinhardt, 2001) stated that for the pre-service and novice teachers especially, they provide incomplete, error-prone and unrelated explanations. One reason for this may be that, novice teachers had taken courses in Calculus where they had frequently encountered the concept of undefined in the topic of limit and derivative. These teachers had the tendency to think of number/zero=infinite. Moreover, some novice teachers gave a numerical result. Tsamir, Ruth, and Tirosh (2000) noted that students may have an intuitive belief that all mathematical operations should have a numerical result.

In comparing the responses according to educational status, teachers with a master's degree suggested more conceptual-based explanations than teachers in license degree. This indicates that the master degree programs did impact teachers' specialized content knowledge positively. Similarly, license degree teachers gave more rule-based and incorrect explanations than master's degree teachers. Previous studies have reported that education status may positively affect the teachers' knowledge (Goldhaber & Brewer, 1996).

CONCLUSION

In the study, it was determined that most of the teachers know the division by zero to be undefined/undetermined. However, their SCK about division by zero was weak. Most of their explanations was rule-based or inadequate. Moreover, they did not know why the division by zero was undefined/undetermined.

It was also found that the experienced teachers provided more conceptual-based explanations than novice teachers. This conclusion shows that the experience is a factor for teachers' SCK.

The results of this study showed that when the teachers' education is higher, their SCK also increased and became varied. In this study, teachers with master's degree gave more conceptual-based explanations about division by zero than teachers with just an undergraduate degree.

EDUCATIONAL IMPLICATIONS

The results of this study showed that although some teachers could give meaningful explanations about division by zero, most of their explanations were narrow. In this context, the reasons of these narrow explanations can lead to further research. The results also show that novice teachers provide more rule-based explanations on division by zero. In this context, the effects of the content and method courses that pre-service teachers have on these explanations need to be examined in depth. Moreover, it is necessary to make some changes in courses about teachers' pedagogical content knowledge in teacher education programs. The results of this study indicate that when the educational status of teachers increase, their meaningful explanations increase, too. For that reason, more opportunities should be given to teachers in order to increase their educational status.

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