

Lifetime Employment and Endogenous Timing in a Mixed Duopoly with Profit-Maximising and Joint-Stock Firms

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Abstract: *This paper investigates endogenous timing in a mixed duopoly consisting of a profit-maximising firm and a joint-stock firm. There are two stages and the firms simultaneously and independently announce in which stage they will offer lifetime employment as a strategic commitment. If both firms decide to offer lifetime employment in the same stage, a simultaneous commitment game occurs, whereas if both firms choose different stages, a sequential commitment game arises. At the end of the game, each firm simultaneously and independently chooses its actual output. The paper presents the equilibrium of the endogenous-timing mixed duopoly model.*

Keywords: Endogenous Timing, Profit-Maximising Firm, Joint-Stock Firm, Lifetime Employment, Strategic Commitment.

JEL classification: C72, D21, L20

Article Received: 20 September 2012; Article Accepted: 23 April 2014

1. Introduction

Recent studies have examined models that endogenously determine the role of a leader and follower. For example, in their pioneering study, Hamilton and Slutsky (1990) examine the novel issue of endogenous timing in two-player games with important modelling implications for several models in industrial economics. In a preplay stage, players decide whether to select actions in the basic game at the first opportunity or to wait until they observe their rivals' first period actions. In one extended game, players first decide when to select actions without committing to actions in the basic game. The equilibrium has a simultaneous play subgame unless payoffs in a sequential play subgame Pareto dominate those payoffs. In another extended game, deciding to select at the first turn requires committing to an action. They show that both sequential play outcomes achieve equilibria only in undominated strategies. Amir (1995) takes the study by Hamilton and Slutsky on endogenous timing (with observable delay) further by showing, via counterexample, that monotonicity of the best-

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response functions in a two-player game is not sufficient to derive predictions about the order of moves and this requires, additionally, the monotonicity of each payoff in the other player's actions. Lu and Poddar (2009) examine a two-stage game of endogenous timing with observable delay in the context of sequential capacity and quantity choice demonstrating that in mixed duopoly, the public and the private firm choose capacity and quantity sequentially in all possible equilibria and no simultaneous capacity or quantity choice case can be a part of an equilibrium. There are many subsequent further studies on this (e.g. see Deneckere and Kovenock, 1992; Furth and Kovenock, 1993; Sadanand and Sadanand, 1996; Lambertini, 1996, 2000; Matsumura, 1999; Van Damme and Hurkens, 1999; Von Stengel, 2010; Ohnishi, 2012). However, these studies do not factor in the participation of joint-stock firms.

Therefore, we investigate endogenous timing in a mixed duopoly consisting of a profit-maximising firm and a joint-stock firm. Only a few studies consider joint-stock firms. For example, Meade (1972) shows the differences in incentives, short-run adjustment, and so forth among profit-maximising, labour-managed and joint-stock firms. Hey (1981) restricts attention to the case of a perfectly competitive firm producing a single output with two inputs, labour and capital, and examines the behaviour of profit-maximising, labour-managed and joint-stock firms. Ohnishi (2010) shows the equilibrium outcome of two-stage Cournot duopoly competition with a profit-maximising firm and a joint-stock firm and finds that the introduction of lifetime employment into the analysis of Cournot mixed competition works as an incentive only for the joint-stock firm.

We examine a mixed market model in which a profit-maximising firm and a joint-stock firm compete against each other. We consider the following situation. There are two stages: stage 1 and stage 2. The firms simultaneously and independently announce in which stage they will offer lifetime employment as a strategic commitment.¹ In the first stage, the firm choosing stage 1 can offer lifetime employment in this stage. In the second stage, the firm choosing stage 2 can offer lifetime employment in this stage. If both firms decide to offer lifetime employment in the same stage, a simultaneous commitment game occurs and if both firms choose different stages, a sequential commitment game arises. At the end of the game, each firm simultaneously and independently chooses its actual output. We discuss the equilibrium of the endogenous-timing mixed duopoly model by developing a theory of duopolistic competition between a profit-maximising firm and a joint-stock firm.

The remainder of this paper is organised as follows. Section 2 describes the model in detail while Section 3 provides supplementary explanations of the model. Section 4 discusses the equilibrium of the model while Section 5 concludes the paper highlighting important findings. Finally, the appendix provides formal proofs and mathematical evidence.

2. The model

There is a market composed of one profit-maximising capitalistic firm (firm P) and one joint-stock income-per-capital-maximising firm (firm J), producing perfectly substitutable goods. In this paper, subscripts P and J denote firm P and firm J respectively and when i and j are used to refer to firms in an expression, they should be understood to refer to P and J with $i \neq j$. There is no possibility of entry or exit. The inverse demand function is represented by, $p = a - Q$, where $Q = q_p + q_j$ and $a > Q$.

The timing of the game is as follows. At the beginning of the game, each firm simultaneously and independently chooses, $t_i \in (1,2)$ where t_i indicates when to offer lifetime employment. That is, $t_i=1$ implies that firm i can offer lifetime employment in stage 1, and $t_i=2$ implies that it can offer lifetime employment in stage 2. Each firm observes t_p and t_j . In stage 1, firm i choosing $t_i=1$ is allowed to offer lifetime employment in this stage. In stage 2, firm i choosing $t_i=2$ is allowed to offer lifetime employment in this stage. If firm i offers lifetime employment, then it chooses an output level $q_i^* > 0$ and enters into a lifetime employment contract with the number of employees necessary to achieve q_i^* . If the firms decide to offer lifetime employment in the same stage, they simultaneously and independently choose output levels q_p^* and q_j^* . At the end of the game, each firm simultaneously and independently chooses its actual output q_i .

Therefore, firm P's profit is given by

$$\pi_p = \begin{cases} pq_p - w_p q_p^2 - r_p q_p^2 - f_p & \text{if } q_p > q_p^* \\ pq_p - w_p q_p^{*2} - r_p q_p^2 - f_p & \text{if } q_p \leq q_p^* \end{cases} \quad (1)$$

where $w > 0$ denotes the wage rate, $r > 0$ is the capital cost rate, and $f > 0$ is the fixed cost.

Firm J's income per capital is given by

$$\phi_j = \begin{cases} \frac{pq_j - w_j q_j^2 - f_j}{k_j} & \text{if } q_j > q_j^* \\ \frac{pq_j - w_j q_j^{*2} - f_j}{k_j} & \text{if } q_j \leq q_j^* \end{cases} \quad (2)$$

where $k > 0$ is the capital input. We consider the following production function:

$$q_j = \sqrt{k_j} \quad (3)$$

From (2) and (3), we can formulate the objective function of firm J as:

$$\phi_j = \begin{cases} \frac{pq_j - w_j q_j^2 - f_j}{q_j^2} & \text{if } q_j > q_j^* \\ \frac{pq_j - w_j q_j^{*2} - f_j}{q_j^2} & \text{if } q_j \leq q_j^* \end{cases} \quad (4)$$

If firm i offers lifetime employment, the cost of $w_i q_i^{*2}$ is sunk. Therefore, if $q_i < q_i^*$, since firm i employs extra employees, firm i has to bear the extra cost of $w_i (q_i^* - q_i)$, and thereby social welfare falls. This paper analyses the subgame perfect Nash equilibrium of the endogenous-timing mixed duopoly model.

3. Supplementary explanations

In this section, we provide supplementary explanation of the model described in the previous section. First, we derive the following reaction functions from (1) and (4):

$$R_p(q_j) = \begin{cases} \frac{a - q_j}{2(1 + w_p + r_p)} & \text{if } q_p > q_p^* \\ q_p^* & \text{if } q_p = q_p^* \\ \frac{a - q_j}{2(1 + r_p)} & \text{if } q_p < q_p^* \end{cases} \quad (5)$$

$$R_j(q_p) = \begin{cases} \frac{2f_j}{a - q_p} & \text{if } q_j > q_j^* \\ q_j^* & \text{if } q_j = q_j^* \\ \frac{2(w_j q_j^{*2} + f_j)}{a - q_p} & \text{if } q_j < q_j^* \end{cases} \quad (6)$$

Firm P's reaction functions are downward sloping, while firm J's reaction functions are upward sloping.

Second, we present the following lemma, which provides a characterisation of lifetime employment as a strategic commitment.

Lemma 1.

If firm i offers lifetime employment and an equilibrium is achieved, then in equilibrium $q_i = q_i^*$

Lemma 1 means that in equilibrium, firm i does not employ extra employees.

Third, we consider each firm's Stackelberg leader output. Firm i selects q_i , and firm j selects q_j after observing q_i . If firm P is the Stackelberg leader, then it maximises its profit $\pi_p(q_p, R_j(q_p))$ with respect to q_p . In addition, if firm J is the leader, then it maximises its income per capital $\phi_j(q_j, R_p(q_j))$ with respect to q_j . We present the following lemma:

Lemma 2.

- (i) Firm P's Stackelberg leader output is lower than its Cournot output.
 - (ii) Firm J's Stackelberg leader output is higher than its Cournot output.
- Lemma 2 indicates that firm P has an incentive to decrease its output while firm J has incentives to increase its output.

4. Equilibrium

In this section, we analyse the equilibrium of the model formulated in Section 2. We discuss the following three cases:

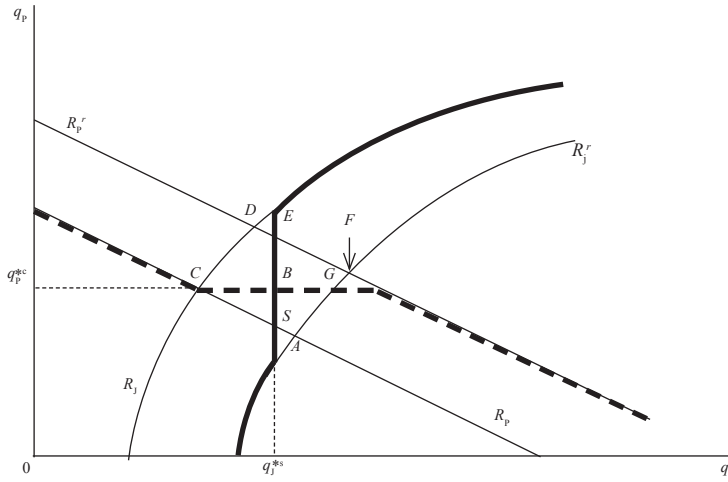
- Case 1. $t_j = 1$ and $t_p = 2$.
- Case 2. $t_j = 2$ and $t_p = 1$.
- Case 3. $t_j = t_p = 1$ ($t_j = t_p = 2$).

We discuss these cases in order.

- Case 1. $t_j = 1$ and $t_p = 2$.

In this case, first firm J moves, then firm P observes firm J's move and subsequently firm P moves. In stage 1, firm J can offer lifetime employment and in stage 2 firm P can offer lifetime employment. At the end of the game, each firm simultaneously and independently chooses q_i , and both firm J's income per capital and firm P's profit are decided.

Figure 1: Reaction curves in the quantity space



The firms' reaction curves are shown in Figure 1. R_i is the reaction curve representing the best quantity choice of firm i in relation to the quantity sold by firm j , if it has to incur the full marginal costs of producing any given quantity; that is, if lifetime employment has not yet been offered. R_i^r is the reaction curve of firm i when lifetime employment has already been offered.

In stage 1, firm J is allowed to offer lifetime employment. Firm J can select a point on segment AC . Firm J's income per capita is the highest at the Stackelberg leader point S on AC . Therefore, if firm J chooses q_j^{*s} corresponding to S and offers lifetime employment, then its reaction curve has a flat segment at q_j^{*s} and becomes the kinked bold lines drawn in this figure.

In stage 2, firm P is allowed to offer lifetime employment. Firm P can select a point on segment EBS , and its profit is the highest at S . At the end of the game, each firm simultaneously and independently chooses q_i corresponding to S , and both firm J's income per capita and firm P's profit are decided. We now state the following proposition:

Proposition 1: Suppose that $t_j = 1$ and $t_p = 2$. Then the equilibrium coincides with the solution where firm J acts as the leader. At equilibrium, firm J's income per capita is higher than in the game with no lifetime employment, while firm P's profit is lower than in the game with no lifetime employment.

Case 2. $t_j = 2$ and $t_p = 1$.

In this case, firm P can move first inter-temporally. In stage 1, firm P can offer lifetime employment, and in stage 2 firm J can offer lifetime employment.

At the end of the game, each firm simultaneously and independently chooses q_p , and both firm J's income per capital and firm P's profit are decided.

In stage 1, firm P is allowed to offer lifetime employment. Firm P can select a point on segment CD . Firm P's profit is the highest at point C on CD . Therefore, if firm P chooses q_p^{*c} corresponding to C and offers lifetime employment, then its reaction curve has a flat segment at q_p^{*c} and becomes the kinked bold broken lines seen in Figure 1.

In stage 2, firm J is allowed to offer lifetime employment. Firm J can select a point on segment CBG , and its profit is the highest at C . At the end of the game, each firm simultaneously and independently chooses q_i^c corresponding to C , and both firm J's income per capital and firm P's profit are decided. The equilibrium outcome can be stated as follows:

Proposition 2: Suppose that $t_j = 2$ and $t_p = 1$. Then the equilibrium coincides with the Cournot-Nash solution with no lifetime employment.

Case 3. $t_j = t_p = 1$ ($t_j = t_p = 2$).

In this case, both firms act in the same stage. If each firm chooses $t_p = 1$, the timing is as follows. In stage 1 each firm can independently offer lifetime employment, and in stage 2 neither firm acts. At the end of the game, each firm simultaneously and independently chooses q_i and both firm J's income per capital and firm P's profit are decided.

In this case, the equilibrium can occur at a point of region $ACDF$. Firm J's income per capital is the highest at S , and firm P's profit is the highest at C . Let us suppose firm J chooses q_j^{*s} corresponding to S and firm P chooses q_p^{*c} corresponding to C , then firm J's reaction curve becomes the kinked bold lines while firm P's reaction curve becomes the kinked bold broken lines. Both firms' reaction curves intersect at B . However, B is not desirable for both firms. A little decrease in q_j^* increases firm J's income per capital. In addition, a little decrease q_p^* increases firm P's profit. The equilibrium outcome can be stated as follows:

Proposition 3: Let us suppose that both firms act in the same stage. Then there exist two equilibria: (i) firm J's unilateral offer equilibrium and (ii) firm P's unilateral offer equilibrium. In (i), firm J's income per capital is higher than in the game with no lifetime employment, and firm P's profit is lower than in the game with no lifetime employment. On the other hand, the equilibrium of (ii) coincides with the Cournot solution with no lifetime employment.

The equilibrium outcomes of three cases are summarised as follows. If firm J acts as the leader, then firm J's income per capital is higher than in the game with no lifetime employment, while firm P's profit is lower than in the game with no lifetime employment. If firm P acts as the leader, then the equilibrium

coincides with the Cournot-Nash solution with no lifetime employment. If both firms act simultaneously, then outcomes can be in equilibrium.

The main result of this study is described by the following proposition.

Proposition 4: The endogenous-timing mixed duopoly model has three equilibria: (i) $t_j = t_p = 1$; (ii) $t_j = 1$ and $t_p = 2$; (iii) $t_j = 2$ and $t_p = 1$. Proposition 4 states that there are both simultaneous and sequential move equilibria in the endogenous-timing mixed duopoly model.

5. Conclusion

We have examined endogenous timing in a mixed duopoly consisting of a profit-maximising firm and a joint-stock firm. At the beginning of the game, each firm simultaneously and independently announces in which stage it will offer lifetime employment as a strategic commitment. In the first stage, the firm choosing stage 1 can offer lifetime employment in this stage. In the second stage, the firm choosing stage 2 can offer lifetime employment in this stage. At the end of the game, each firm simultaneously and independently chooses its actual output. We have shown that there exist both simultaneous and sequential move equilibria in this endogenous-timing mixed duopoly model.

Note

¹ For details of lifetime employment as a strategic commitment, see Ohnishi (2001, 2002).

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Appendices

We begin by proving Lemmas 1 and 2.

Proof of Lemma 1

We prove that if firm P offers lifetime employment and an equilibrium is achieved, then in equilibrium $q_p = q_p^*$. Consider the possibility that $q_p < q_p^*$ in equilibrium. From (1), if $q_p < q_p^*$, firm P must employ extra employees if necessary to produce $q_p^* - q_p$. That is, firm P can increase its profit by reducing q_p^* , and the equilibrium point does not change in $q_p \leq q_p^*$. Hence, $q_p < q_p^*$ does not result in an equilibrium.

Consider the possibility that $q_p > q_p^*$ in equilibrium. From (2), we see that firm P has to incur the full marginal costs of producing any given quantity. It is impossible for firm P to change its output in equilibrium because such a strategy is not credible. That is, if $q_p > q_p^*$, lifetime employment does not function as a strategic commitment.

The proof of offer by firm J is omitted since it is essentially the same as the proof of offer by firm P. Q.E.D.

Proof of Lemma 2

- (i) Firm P selects q_p , and firm J selects q_j after observing q_p . That is, firm P maximises $\pi_p(q_p, R_j(q_p))$ with respect to q_p . Therefore, firm P's Stackelberg leader output satisfies the first-order condition:

$$\frac{\partial \pi_p}{\partial q_p} + \frac{\partial \pi_p}{\partial q_j} \frac{\partial R_j}{\partial q_p} = 0, \quad (7)$$

where $\partial \pi_p / \partial q_j = -q_p$ is negative, while $\partial R_j / \partial q_j$ is positive from (6). To satisfy (7), $\partial \pi_p / \partial q_p$ must be positive. Thus, firm P's Stackelberg leader output is smaller than its Cournot output.

- (ii) This proof is omitted since it is essentially the same as the proof of (i). Q.E.D.

We now prove the propositions:

Proof of Proposition 1

In stage 1, firm J is allowed to offer lifetime employment. Lemma 2 (ii) shows that firm J's Stackelberg leader output is higher than its Cournot output. From (6), we see that firm J's income-per-capital-maximising output is higher when firm J offers lifetime employment than when it does not. Lemma 1 shows that in equilibrium $q_j = q_j^*$. Hence, firm J can increase its income per capital by offering lifetime employment. Therefore, firm J chooses q_j^{*s} corresponding to its optimal output level and offers lifetime employment.

If firm J offers lifetime employment, then its reaction function has a flat segment at q_j^{*s} level. That is, firm J's reaction function has a zero slope at $q_j = q_j^{*s}$. This implies that even if q_p is increased, q_j is constant. In stage 2, firm P can offer lifetime employment. From (5), we see that firm P's profit-maximising output is higher when firm P offers lifetime employment than when it does not. Furthermore, $\pi_p = pq_p - w_p q_p^2 - r_p q_p^2 - f_p$ is continuous and concave. A little increase in firm P's output does not change firm J's output and decreases firm P's profit. That is, the offer of lifetime employment by firm P decreases its own profit. Our equilibrium concept is the subgame perfect Nash equilibrium, and all information in the model is common knowledge. Therefore, firm J can always influence firm P to offer lifetime employment by choosing the appropriate level of q_j^* . Thus, the equilibrium coincides with the solution where firm J is the leader.

Next, we prove that at equilibrium, firm P's profit is lower than in the game with no lifetime employment. The offer of lifetime employment by firm J increases its output. Since $\partial \pi_p / \partial q_j = -q_p < 0$, increasing q_j decreases π_p given q_p . Thus, Proposition 1 follows. Q.E.D.

Proof of Proposition 2

In stage 1, firm P can offer lifetime employment. Lemma 2 (i) shows that firm P's Stackelberg leader output is higher than its Cournot output. However, from (5), we see that firm P's profit-maximising output is higher when firm P offers lifetime employment than when it does not. Hence, firm P cannot increase its profit by offering lifetime employment. In stage 2, firm J can offer lifetime employment. Lemma 2 (ii) shows that firm J's Stackelberg leader output is higher than its Cournot output. From (6), we see that firm J's income-per-capital-maximising output is higher when firm J offers lifetime employment than when it does not. Lemma 1 shows that in equilibrium $q_j = q_j^*$. Hence, if firm P does not offer lifetime employment, then firm J offers lifetime employment. From Proposition 1, we see that if firm J unilaterally offers lifetime employment, then its income per capital is higher than in the game with no lifetime employment, while firm P's profit is lower than in the game with no lifetime employment. If firm P does not offer lifetime employment in stage 1, then its profit is lower than in the game with no lifetime employment.

If firm P chooses q_p^{*c} corresponding to the Cournot solution with no lifetime employment and offers lifetime employment, then its reaction function has a flat segment at q_p^{*c} level. In stage 2, firm J can offer lifetime employment. Firm J's income-per-capital-maximising output is higher when firm J offers lifetime employment than when it does not. Furthermore, $\phi_j = (pq_j - w_j q_j^2 - f_j) / q_j^2$ is continuous and concave. A little increase in firm J's output does not change firm P's output but decreases firm J's income per capital. That is, the offer of lifetime employment by firm J decreases its own income per capital. Our equilibrium concept is the subgame perfect Nash equilibrium, and all information in the model is common knowledge. Therefore, firm P can always influence firm J to offer lifetime employment by choosing the appropriate level of q_p^* . Thus, firm P chooses q_p^{*c} corresponding to the Cournot solution with no lifetime employment and offers lifetime employment. Q.E.D.

Proof of Proposition 3

In this case, both firms are allowed to offer lifetime employment in the same stage. First, we consider the case in which firm P unilaterally offers lifetime employment. Lemma 2 (i) shows that firm P's Stackelberg leader output is higher than its Cournot output. However, from (5), we see that firm P's profit-maximising output is higher when firm P offers lifetime employment than when it does not. Hence, firm P cannot increase its profit by offering lifetime employment. Therefore, if firm P unilaterally offers lifetime employment, then the equilibrium coincides with the Cournot solution with no lifetime employment.

Second, we consider the case in which firm J unilaterally offers lifetime employment. Lemma 2 (ii) shows that firm J's Stackelberg leader output is higher than its Cournot output. From (6), we see that firm J's income-per-capital-maximising output is higher when firm J offers lifetime employment than when it does not. Lemma 1 shows that in equilibrium $q_j = q_j^*$. From Proposition 1, we see that if firm J unilaterally offers lifetime employment, then firm J's income per capital is higher than in the game with no lifetime employment, while firm P's profit is lower than in the game with no lifetime employment.

Third, we consider the case in which both firms offer lifetime employment. Lemma 2 (i) shows that firm P's Stackelberg leader output is higher than its Cournot output. However, from (5), we see that firm P's profit-maximising output is higher when firm P offers lifetime employment than when it does not. Hence, firm P cannot increase its profit by offering lifetime employment. Therefore, firm P chooses q_p^{*c} corresponding to the Cournot solution with no lifetime employment. Lemma 2 (ii) shows that firm J's Stackelberg leader output is higher than its Cournot output. From (6), we see that firm J's income-per-capital-maximising output is higher when firm J offers lifetime employment than when it does not. Lemma 1 shows that in equilibrium $q_j = q_j^*$. Hence, firm J can increase its income per capital by offering lifetime employment. Therefore, firm J chooses q_j^{*s} corresponding to its optimal output level and offers lifetime employment. Each firm's reaction function has a zero slope at $q_i = q_i^*$. This implies that even if q_j is increased, q_i is constant. Firm J maximises its income per capital by decreasing q_j and q_j^* to a point of R_j , and firm P maximises its profit by decreasing q_p and q_p^* to a point of R_p .

We can now consider the following matrix:

		Firm P	
		Commitment	No Commitment
Firm J	Commitment	ϕ_j^B, π_p^B	ϕ_j^J, π_p^J
	No Commitment	ϕ_j^P, π_p^P	ϕ_j^N, π_p^N

From the preceding results, we see that $\phi_j^B < \phi_j^P = \phi_j^N < \phi_j^J, \pi_p^J < \pi_p^P = \pi_p^N$, and $\pi_p^B < \pi_p^J$. Thus, this case has two equilibria: (No commitment, Commitment) and (Commitment, No commitment). Q.E.D.

Proof of Proposition 4

At the beginning of the game, each firm simultaneously and independently chooses $t_i \in (1, 2)$. Each firm observes t_p and t_j . In stage 1, firm i choosing $t_i = 1$ is allowed to offer lifetime employment in this stage. In stage 2, firm i

choosing $t_i = 2$ is allowed to offer lifetime employment in this stage. At the end of the game, each firm simultaneously and independently chooses q_p and both firm J's income per capital and firm P's profit are decided. Our equilibrium concept is the subgame perfect Nash equilibrium, and all information in the model is common knowledge. Therefore, firm J can always influence firm P to offer lifetime employment by choosing the appropriate level of q_j^* .

From Propositions 1 to 3, we can consider the following two matrices:

(I)			Firm P	
			Stage 1	Stage 2
	Firm J	Stage 1	ϕ_J^L, π_P^F	ϕ_J^L, π_P^F
		Stage 2	ϕ_J^C, π_P^C	ϕ_J^L, π_P^F

(II)			Firm P	
			Stage 1	Stage 2
	Firm J	Stage 1	ϕ_J^C, π_P^C	ϕ_J^L, π_P^F
		Stage 2	ϕ_J^C, π_P^C	ϕ_J^C, π_P^C

Here, $\phi_J^C < \phi_J^L$ and $\pi_P^F < \pi_P^C$. In (I), the equilibrium is both (Stage 1, Stage 1) and (Stage 1, Stage 2). In (II), the equilibrium is both (Stage 1, Stage 1) and (Stage 2, Stage 1). Q.E.D.